DECOMPOSITION OF DIRECT IMAGES OF LOGARITHMIC DIFFERENTIALS

DONU ARAPURA

In [Ko], Kollár proved a remarkable theorem that given a projective map $f : X \to Y$ from a smooth complex algebraic variety to an arbitrary variety, the derived direct image of the canonical sheaf $\mathbb{R}f_*\omega_X$ decomposes as a sum $\bigoplus R^i f_*\omega_X[-i]$. So in particular, the Leray spectral sequence for ω_X degenerates. Saito [S] gave a second proof using his theory of Hodge modules, which gives a good conceptual explanation for the theorem, in spite of the heavy prerequisites. A third analytic proof was given by Takegoshi [T]. The purpose of this note is to give another proof, which is fairly short. The basic idea is as follows. The weak semistable reduction theorem of Abramovich and Karu [AK], can be interpreted as saying that any map can be converted, in the appropriate sense, to a map which is particularly nice from the point of view of logarithmic geometry [K2]. Since we would rather not assume knowledge of this area, we start with the semistable case, which contains all the main ideas while avoiding the language of log structures. The principal result here is that the direct images of the sheaves of logarithmic differentials $\mathbb{R}f_*\Omega^k_{X/Y}(\log E)$ decompose as above. It is important that we work with all degrees k simultaneously, because the sum of these direct images carries a Lefschetz operator. The proof of the decomposition theorem ultimately comes down to checking that this sum satisfies the hard Lefschetz theorem and then applying Deligne's theorem [D]. In the second section, this result is refined to the case of sufficiently nice log smooth maps, i.e. maps of the sort that come up after the weak semistable reduction theorem is applied. The previous arguments carry over almost verbatim. The final step is to show that this refined theorem implies Kollár's.

We work over $\mathbb{C}$. The words morphism and map are used interchangeably, according to whim.

1. Semistable case

By a *log pair*, we will mean a pair consisting of a smooth scheme X and a divisor $E \subset X$ with normal crossings. As we will explain later, the set of such pairs embeds into a fairly natural category, but for the moment, we just describe the nicest morphisms. A *semistable* morphism of log pairs $f : (X, E) \to (Y, D)$ is a morphism f of schemes which is given étale locally (or analytically) by

$$y_1 = x_1 x_2 \dots x_{r_1}$$
$$y_2 = x_{r_1+1} \dots x_{r_2}$$
$$\dots$$
$$y_{d+1} = x_{r_d+1}$$
$$\dots$$

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such that $x_1 \dots x_{r_d}$ and $y_1 \dots y_d$ are the local equations for E and D respectively. Such maps are flat and the restriction $X - E \to Y - D$ is smooth. We observe also that $E = f^*D$ as a divisor. Given a log pair $\mathcal{X} = (X, E)$, set

$$\Omega^k_{\mathcal{X}} = \Omega^k_X(\log E)$$

which is generated locally by k-fold wedge products of

$$\frac{dx_1}{x_1}, \dots \frac{dx_{r_d}}{x_{r_d}}, dx_{r_d+1}, \dots$$

It is clear that these sheaves behave "functorially" with respect to semistable morphisms, and it is not difficult to the check that the sheaf of relative log differentials, defined by the exact sequence

$$0 \to f^*\Omega^1_{\mathcal{Y}} \to \Omega^1_{\mathcal{X}} \to \Omega^1_{\mathcal{X}/\mathcal{Y}} \to 0, \tag{1}$$

is locally free. The sheaf $\Omega^k_{\mathcal{X}/\mathcal{Y}}$ is the k-th exterior power of $\Omega^1_{\mathcal{X}/\mathcal{Y}}$.

Lemma 1.1. *If $f : (X, E) \to (Y, D)$ is semistable of relative dimension d, then*

$$\Omega^d_{(X,E)/(Y,D)} \cong \omega_{X/Y}$$

where $\omega_{X/Y}$ is the relative dualizing sheaf.

Proof. After taking the top exterior power of (1), we obtain

$$\Omega^d_{(X,E)/(Y,D)} \cong \omega_X(E) \otimes f^*\omega_Y(D)^{-1} \cong \omega_{X/Y}$$

□

Before describing the main result, we make a few remarks about the analytic version of this story. Everything we said above can be repeated when smooth schemes are replaced by complex manifolds. In addition, there is a new construction often called the *real blow up*. Given an analytic log pair $\mathcal{X}$, i.e. a pair consisting of complex manifold X together with a normal crossing divisor E, the real blow up is a topological space $\mathcal{X}^{log}$ with a continuous map $\lambda : \mathcal{X}^{log} \to X$ [KN, K1]. When $\mathcal{X} = (\mathbb{C}^n, \{z_1 \dots z_n = 0\})$, $\mathcal{X}^{log} = ([0, \infty) \times S^1)^n$ with $\lambda(r_1, u_1, r_2, u_2, \dots) \mapsto (r_1u_1, r_2u_2, \dots)$. A similar description holds when $\mathbb{C}^n$ is replaced by a polydisk. These local blow ups can be patched together for arbitrary X. In fact the real blow up can be made completely canonical, so that a semistable map $\mathcal{X} \to \mathcal{Y}$ induces a continuous map $f^{log} : \mathcal{X}^{log} \to \mathcal{Y}^{log}$. One can picture $(X, E)^{log}$ as adding an ideal boundary to $X - E$ which is homeomorphic to the boundary of a tubular neighbourhood about E, and this picture is compatible with what is happening on Y. This leads to the remarkable fact, due to Usui, that f^{log} is topologically a fibre bundle which lies over f; see [NO] for an overview and refinement needed later on. We note also that $\mathcal{X}^{log}$ can be made into a ringed space with structure sheaf $\mathcal{O}_{\mathcal{X}^{log}}$ such that λ becomes a morphism.

Theorem 1.2. *Let $f : \mathcal{X} \to \mathcal{Y}$ be a projective semistable map of log pairs over $\mathbb{C}$, then*

$$\mathbb{R}f_*\Omega^k_{\mathcal{X}/\mathcal{Y}} \cong \bigoplus_i R^if_*\Omega^k_{\mathcal{X}/\mathcal{Y}}[-i]$$

for all k. Moreover the same result holds for a proper holomorphic semistable map of analytic log pairs provided that there is a 2-form on X which restricts to a Kähler form on the components of all the fibres.

Before giving the proof, we record the following elementary fact, which can be proved by induction on the length of the filtrations.

Lemma 1.3. *Suppose that $(V_i, F_i^\bullet)$ are two filtered finite dimensional vector spaces such that $\dim Gr^p(V_1) = \dim Gr^p(V_2)$ for all p. Then a linear isomorphism $L : V_1 \to V_2$ which is compatible with the filtrations will induce an isomorphism of associated graded spaces.*

Proof of theorem. Let d denote the relative dimension of f. When f is algebraic, by the work of Illusie [I, cor 2.6], the sheaves $R^i f_* \Omega^k_{\mathcal{X}/\mathcal{Y}}$ are locally free and commute with base change, and the relative Hodge to de Rham spectral sequence degenerates at E_1. The proof, which uses a refinement of the Deligne-Illusie method, is quite short. A Hodge theoretic proof of all of these results, which works in the Kähler case, was given by Fujisawa [F, thm 6.10]. Let $V^k = \mathbb{R} f_* \Omega^k_{\mathcal{X}/\mathcal{Y}}$ and $V = \bigoplus_k V^k[-k]$. Let $\eta' \in H^1(X, \Omega^1_X)$ denote either c_1 of a relatively ample line bundle or the class of a relative Kähler form, and let $\eta \in H^1(\Omega^1_{\mathcal{X}/\mathcal{Y}}) \cong Hom_{D(Y)}(\mathcal{O}_Y, \mathbb{R} f_* \Omega^1_{\mathcal{X}/\mathcal{Y}}[1])$ denote the image of η'. We have a Lefschetz operator $L : V^k \to V^{k+1}[1]$ given by cup product with η. By adding these, we get a map $L : V \to V[2]$. If we can establish the hard Lefschetz property that L^i induces

$$L^i : \mathcal{H}^{d-i}(V) \cong \mathcal{H}^{d+i}(V) \tag{2}$$

for all i, then the theorem will follow from Deligne's theorem [D]. Note that we have canonical isomorphisms

$$\mathcal{H}^i(V)_y \cong \bigoplus_k (R^{i-k} f_* \Omega^k_{\mathcal{X}/\mathcal{Y}})_y \cong \bigoplus_k (Gr^k_F \mathbb{R}^i f_* \Omega^\bullet_{\mathcal{X}/\mathcal{Y}})_y \tag{3}$$

where F is the Hodge filtration. This isomorphism respects the action by L. We note also that the ranks of $R^{d-i-k} f_* \Omega^k_{\mathcal{X}/\mathcal{Y}}$ and $R^{d-k} f_* \Omega^{k+i}_{\mathcal{X}/\mathcal{Y}}$ coincide with the ranks of $R^{d-i-k} f_* \Omega^k_{X-E/Y-D}$ and $R^{d-k} f_* \Omega^{k+i}_{X-E/Y-D}$ respectively, and therefore with each other. Therefore by lemma 1.3 and Nakayama's lemma, it suffices to check the hard Lefschetz property for $(\mathbb{R}^i f_* \Omega^\bullet_{\mathcal{X}/\mathcal{Y}})_y$ and all $y \in Y$. By [K1] , we have a canonical identification

$$\lambda^* \mathbb{R} f_* \Omega^\bullet_{\mathcal{X}/\mathcal{Y}} \cong \mathbb{R} f^{log}_* \mathbb{C} \otimes \mathcal{O}_{Y^{log}} \tag{4}$$

Note that $\mathbb{R} f^{log}_* \mathbb{C}$ also has an action by L, and (4) respects these actions. Since the stalk $(R^i f^{log}_* \mathbb{C})_y$ is just $H^i(X_y, \mathbb{C})$, when $y \notin D$, and $R^i f^{log}_* \mathbb{C}$ is a local system, we can conclude that we have a hard Lefschetz theorem everywhere, i.e. $L^i : R^{d-i} f^{log}_* \mathbb{C} \cong R^{d+i} f^{log}_* \mathbb{C}$. By the previous remarks, this implies (2). □

Corollary 1.4 (Kollár)**.** *If f is semistable, then*

$$\mathbb{R} f_* \omega_X \cong \bigoplus_i R^i f_* \omega_X[-i]$$

Proof. By lemma 1.1 and the above theorem,

$$\mathbb{R} f_* \omega_{X/Y} \cong \bigoplus_i R^i f_* \omega_{X/Y}[-i]$$

Now tensor both sides by ω_Y. □

2. General case

The collection of log pairs and semistable maps is too limited for our purposes. The most convenient framework is the category of log schemes of Fontaine, Illusie and Kato [K2]. In order to motivate this, we start with an example which serves as a bridge between these notions.

Example 2.1. *Given a log pair (X, E), let $M \subset \mathcal{O}_X$ be the subsheaf of functions invertible outside of E. This is a monoid under multiplication, which contains $\mathcal{O}_X^*$. In the reverse direction, E can be recovered from M as the support of $M/\mathcal{O}_X^*$.*

We now recall the basic definitions and results. The details can be found in [K2, O]. A *pre-log structure* on a scheme X is a sheaf of commutative monoids M on the étale topology X_{et} together with a homomorphism $\alpha : M \to \mathcal{O}_X$, where the latter is treated as a monoid with respect to multiplication. It is a *log structure* if $\alpha^{-1}\mathcal{O}_X^* \cong \mathcal{O}_X^*$. A pre-log structure can be completed to a log structure in a natural way. A *log scheme* is a scheme equipped with a log structure. In order to avoid confusion below, we reserve the symbols $\mathcal{X}, \mathcal{Y}, \dots$ for log schemes, and use the corresponding symbols $X, Y, \dots$ for the underlying schemes. We already gave one basic example. Here are some more.

Example 2.2. *Any scheme X can be turned into a log scheme with the* trivial *log structure $M = \mathcal{O}_X^*$.*

Example 2.3. *Given a commutative monoid M and a commutative ring R. The monoid algebra $R[M]$ can be viewed as the R-algebra spanned by symbols $\{x^m \mid m \in M\}$. Its spectrum* $\operatorname{Spec} R[M]$ *carries a pre-log structure given by the natural map $M \to R[M]$, $m \mapsto x^m$. This can be completed to a log structure, referred as the canonical log structure associated to this data.*

The last example hints at the connection with toric geometry. This can be made stronger by imposing suitable finiteness conditions that M is finitely generated and embeddable into an abelian group as a saturated monoid. If M satisfies these conditions, it is called *fine and saturated* or simply *fs*. Note that there is a canonical choice for the ambient group, namely the group M^{gp} which is given by a Grothendieck type construction. In general, we will restrict our attention to fs log structures which are those étale locally isomorphic to example 2.3, with M fs. The first two examples satisfy this. Indeed, the first is locally $\operatorname{Spec} \mathbb{C}[\mathbb{N}^n]$ with its canonical log structure. A morphism of log schemes consists of a morphism of schemes and a compatible morphisms of monoids. Morphisms of fs log schemes are étale locally modelled as follows.

Example 2.4. *A homomorphism of fs monoids $P \to Q$ and rings $R \to S$ induces a morphism of fs log schemes* $\operatorname{Spec} S[Q] \to \operatorname{Spec} R[P]$.

Log structures give rise to logarithmic differentials in a rather general way. Given a log scheme $\mathcal{X}$ over $\mathbb{C}$, we define the $\mathcal{O}_X$-module $\Omega^1_{\mathcal{X}} = \Omega^1_{\mathcal{X}/\mathbb{C}}$ as the universal sheaf which receives a $\mathbb{C}$-linear derivation $d : \mathcal{O}_X \to \Omega^1_{\mathcal{X}}$ and homomorphism $\operatorname{dlog} : M \to \Omega^1_{\mathcal{X}}$ satisfying $m \operatorname{dlog} m = dm$. Of course, by the universal property of ordinary Kähler differentials, we have a map $\Omega^1_X \to \Omega^1_{\mathcal{X}}$ taking df to df, but this is generally not an isomorphism. There is a relative version of differentials $\Omega^1_{\mathcal{X}/\mathcal{Y}}$ for a morphism $\mathcal{X} \to \mathcal{Y}$ of log schemes. There is a notion of smoothness in this setting. A morphism is *log smooth* if it is étale locally given by $\operatorname{Spec} S[Q] \to \operatorname{Spec} R[P]$, where $P \to Q$

is a homomorphism such that the kernel and the torsion part of the cokernel of $P^{gp} \to Q^{gp}$ are finite and $\mathrm{Spec}\, S \to \mathrm{Spec}\, R$ is smooth as a map of schemes. (NB: we are making use of the characteristic 0 assumption to simplify the statement.) For such a map $\Omega^1_{\mathcal{X}/\mathcal{Y}}$ and therefore its exterior powers $\Omega^k_{\mathcal{X}/\mathcal{Y}}$, are locally free. Here is a basic example:

Example 2.5. *Suppose that $f : (X, E) \to (Y, D)$ is a semistable map. If $\mathcal{Y}$ and $\mathcal{X}$ are log schemes defined by D and E, then $f : \mathcal{X} \to \mathcal{Y}$ is a log smooth morphism. Furthermore $\Omega^k_{\mathcal{X}/\mathcal{Y}} = \Omega^k_{(X,E)/(Y,D)}$.*

One should be careful about reading too much into the notion of log smoothness; it really amounts to a toroidal condition which is not quite enough for us. For instance, while smoothness implies flatness, the corresponding statement for log smoothness is false as the next example shows.

Example 2.6. *If $P \subset \mathbb{N}^2$ is the submonoid spanned by $(1, 0)$ and $(1, 1)$, the inclusion induces an affine chart of the blow up of $\mathbb{C}^2$ mapping to $\mathbb{C}^2$. This is log smooth but not flat as a map of schemes.*

To rectify the above situation, we need a few more conditions. A morphism of fs log schemes is *exact* if locally it is given as in example 2.4 with $h : P \to Q$ satisfying $(h^{gp})^{-1}(Q) = P$. It is *integral* if $\mathbb{Z}[P] \to \mathbb{Z}[Q]$ is flat. Integral maps are exact and also flat as a map of schemes. Example 2.5 is integral and therefore exact, while 2.6 is neither. Finally, for the sake of expedience, we will say that a log smooth map is *saturated* if it is integral and the scheme theoretic fibres are reduced, cf [IT, 3.64].

We need a few more definitions and remarks before getting to the main theorem. The *characteristic monoids* of a log scheme (X, M) are the stalks $M_x/\mathcal{O}^*_x$; these are *free* if they are isomorphic to $\mathbb{N}^r$ for some r. For instance, example 2.1 has free characteristic monoids. Finally, we note that there is a parallel theory of fs log analytic spaces, and the real blow up is defined for these.

Theorem 2.7. *Let $f : \mathcal{X} \to \mathcal{Y}$ be an exact log smooth map of fs log schemes and suppose that the characteristic monoids of $\mathcal{Y}$ are free. Then*

$$\mathbb{R}f_*\Omega^k_{\mathcal{X}/\mathcal{Y}} \cong \bigoplus_i R^i f_*\Omega^k_{\mathcal{X}/\mathcal{Y}}[-i]$$

for all k.

Proof. As a first step, we use a result of Illusie, Kato and Nakayama [IKN, cor 7.2], in the place of [I], to conclude that sheaves $R^i f_*\Omega^k_{\mathcal{X}/\mathcal{Y}}$ are locally free and that the relative Hodge to de Rham spectral degenerates. From this point on, the proof is identical to the proof of theorem 1.2. We should point out that the results used earlier of Nakayama-Ogus [NO], that $f^{\log}$ is a fibre bundle, and F. Kato [K1], to obtain (4), are still valid under assumptions of the theorem. □

Before turning to Kollár's theorem, we need a small refinement of lemma 1.1.

Lemma 2.8. *Suppose that Y is a smooth variety, $D \subset Y$ is a divisor with normal crossings, X is a variety with normal Gorenstein singularities, and that $f : \mathcal{X} \to \mathcal{Y}$ is log smooth and saturated with respect to the log structures $\mathcal{X}$ defined by $f^{-1}D$ and $\mathcal{Y}$ defined by D. Then $\Omega^d_{\mathcal{X}/\mathcal{Y}} \cong \omega_{X/Y}$, where $d = \dim X - \dim Y$.*

Proof. The line bundle

$$\Omega^d_{\mathcal{X}/\mathcal{Y}} \otimes \omega^{-1}_{X/Y} \cong \mathcal{O}_X(E) \tag{5}$$

where $E = \sum n_i E_i$ is a Cartier divisor supported on $f^{-1}D$. In fact, we can make the choice of E canonical. The divisor E is determined by its restriction to the regular locus of $U \subseteq X$, since the complement of U has codimension at least 2. So we may replace X by U. Then E is the divisor of the canonical map

$$\omega_{X/Y} \cong \Omega^d_X \otimes f^*\omega_Y^{-1} \to \Omega^d_{\mathcal{X}/\mathcal{Y}}$$

Our goal is to show that E is in fact trivial as a divisor. Since this is now a local problem, there is no loss of generality in assuming that Y is affine. The log smoothness and saturation assumptions imply that f is flat. Therefore all of the irreducible components of $f^{-1}D$, and in particular E, must map onto components of D. Thus the preimage of a general curve $C \subset Y$ will meet all the components of E. The conditions of log smoothness and saturation and the isomorphism (5) are stable under base change to C. Therefore we may assume that Y is a curve and that D is a point with local parameter y. The fibre $f^{-1}D$ is reduced because f is saturated. Choose a general point p of an irreducible component E_i of E. Then E_i is smooth in neighbourhood of p and f^*y gives a defining equation for it. Thus we may choose coordinates $x_1 = f^*y, x_2, \ldots x_n$ at p. Then a local generator of $\Omega^d_{\mathcal{X}/\mathcal{Y}}$ at p is given by

$$(d\log x_1 \wedge dx_2 \wedge \ldots \wedge dx_n) \otimes (d\log y)^{-1} = (dx_1 \wedge dx_2 \wedge \ldots \wedge dx_n) \otimes (dy)^{-1}$$

which coincides with a generator of $\omega_{X/Y}$. Thus E is trivial. $\square$

Theorem 2.9 (Kollár)**.** *If X smooth and $f : X \to Y$ is projective then*

$$\mathbb{R}f_*\omega_X \cong \bigoplus_i R^i f_*\omega_X[-i]$$

Proof. We use the weak semistable reduction reduction [AK], which when interpreted in the log setting (cf [IT, 3.10]) yields a diagram

$$\begin{array}{ccc} X' & \xrightarrow{\pi} & X \\ \downarrow{\scriptstyle f'} & & \downarrow{\scriptstyle f} \\ D \subset Y' & \xrightarrow{p} & Y \end{array}$$

where p is generically finite, Y' is smooth, D is a divisor with simple normal crossings, X' is birational to the fibre product, and f' is log smooth and saturated with respect to the log structures $\mathcal{X}'$ defined by $f^{-1}D$ and $\mathcal{Y}'$ defined by D. Finally, it is known that X' has rational Gorenstein singularities.

By lemma 2.8 and theorem 2.7, we obtain

$$\mathbb{R}f'_*\omega_{X'/Y'} = \bigoplus R^i f'_*\omega_{X'/Y'}[-i]$$

and therefore

$$\mathbb{R}f'_*\omega_{X'} = \bigoplus R^i f'_*\omega_{X'}[-i]. \tag{6}$$

Fix a resolution of singularities $g : \tilde{X} \to X'$. Then $\mathbb{R}g_*\omega_{\tilde{X}} = g_*\omega_{\tilde{X}} = \omega_{X'}$, where the first equality follows from the Grauert-Riemenschneider vanishing theorem, and

the second from the fact that X' has rational singularities. This together with (6) shows that

$$\mathbb{R}(f' \circ g)_* \omega_{\tilde{X}} = \bigoplus R^i (f' \circ g)_* \omega_{\tilde{X}}[-i]. \tag{7}$$

We have an inclusion $(\pi \circ g)^* \omega_X \subset \omega_{\tilde{X}}$ which gives an injection

$$\sigma : \omega_X \hookrightarrow (\pi \circ g) * (\pi \circ g)^* \omega_X \hookrightarrow (\pi \circ g)_* \omega_{\tilde{X}}$$

The map σ splits the normalized Grothendieck trace

$$\tau = \frac{1}{\deg X'/X} tr : \mathbb{R}(\pi \circ g)_* \omega_{\tilde{X}} \cong (\pi \circ g)_* \omega_{\tilde{X}} \to \omega_X$$

It follows that ω_X is a direct summand of $(\pi \circ g)_* \omega_{\tilde{X}}$, and that this relation persists after applying a direct image functor. Therefore applying τ to (7) yields

$$\mathbb{R} f_* \omega_X = \bigoplus R^i f_* \omega_X[-i].$$

□

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Department of Mathematics, Purdue University, West Lafayette, IN 47907, U.S.A.

E-mail address: dvb@math.purdue.edu